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Author(s): Praveen K. Kopalle and Donna L. Hoffman
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Generalizing the Sensitivity Conditions in an Overall Index of Product Quality

PRAVEEN K. KOPALLE
DONNA L. HOFFMAN*

The issue of the sensitivity of weighted linear composites to attribute-importance weights has attracted researchers from various disciplines, including marketing, psychology, and statistics. At issue is how sensitive a weighted scale is to a particular choice of weights. Scale sensitivity is defined by a negative correlation between two scales. By considering the general case of *n* attributes and using an algebraic approach, we specify the precise sufficiency conditions under which two scales will correlate negatively and thus be sensitive to the weights chosen. These sensitivity conditions are derived in the context of the computation of an overall product-quality scale, which is simply a special case of the general multiattribute problem. We illustrate these conditions for the case of two quality scales using examples from *Test* (from 1983), a German magazine similar to *Consumer Reports*, and from *Places Rated Almanac* (from 1987).

Using simulation techniques, several researchers (Curry, Louviere, and Augustine 1981; Green, Desarbo, and Kedia 1980, 1981) have examined how sensitive importance weights in multiattribute models are to sample size, number of attributes, and number of levels per attribute. However, sensitivity of the criterion variable to the importance weights in different interattribute correlation environments has not been tested. Johnson, Meyer, and Ghose (1989) deal specifically with a negatively correlated environment but do not address the issue of scale sensitivity to the importance weights. In a recent article, Curry and Faulds (1986) discuss computing an overall product-quality index in a multiattribute framework. At issue is how sensitive a weighted quality scale is to the particular choice of weights. This is important because overall-quality scales are used regularly by consumers making buying decisions and by researchers investigating the relationship between product quality and marketing variables. We define scale sensitivity, following previous research, by a negative correlation between the two quality scales.

Sensitivity of the criterion variable to importance weights (or regression weights in the case of a general linear composite) has also plagued psychologists and statisticians for some time (see Dawes [1979] for a good review). For instance, Schmidt (1971), Einhorn and Hogarth (1975), and Wainer (1976) have shown that in a regression model unit weights performed as well as a differential weighting vector. Using data from *Places Rated Almanac* (Boyer and Savageau 1985), Becker et al. (1987) showed that each of 134 cities can be placed in the first position simply by using different weighting vectors. Because the computation of an overall product-quality index is simply a special case of the general multiattribute problem, our results have direct implications for multiattribute models and, more broadly, for weighted linear composites.

We examine the correlation between two criterion variables (the overall quality scales) with identical predictors (the attributes) and two different sets of importance weights (weighting vectors). This is relevant because there is debate in the consumer-behavior literature about whether quality can be represented by a single scale, such as that reported in *Consumer Reports* (Curry and Faulds 1986; Hjorth-Andersen 1984, 1986; Sproles 1986). Many researchers consider this ranking to be a measure of overall product quality (see, e.g., Archibald, Haulman, and Moody 1983; Reisz 1978; Tellis and Wernerfelt 1987). However, Hjorth-Andersen (1984) questions the concept of overall quality from an empirical perspective. Using Kendall's coefficient of concordance, W, he argues that overall quality as measured and reported in *Consumer Reports* is highly sensitive to the choice of weights.

Curry and Faulds (1986) considered the effect of different weighting schemes in situations involving three

*Praveen K. Kopalle is a doctoral candidate in marketing at the Graduate School of Business, Columbia University, New York, NY 10027. Donna L. Hoffman is associate professor of marketing at the School of Management, University of Texas at Dallas, Richardson, TX 75083. The authors thank David J. Curry for graciously providing access to the data used in this study. Special thanks to Joao Assuncao and the seminar participants at Columbia University for their discussion and many helpful comments on an earlier version of this article.

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attributes and five different interattribute correlation structures. Using the product-moment correlation coefficient as a test of congruence between two quality scales, they demonstrate through numerical illustration and computer simulation that in most cases a weighted scale is actually insensitive to the particular weights chosen. Their results were further supported by empirical examination of data from *Test* (a magazine comparable to *Consumer Reports*) published by the German testing agency, Stiftung Warentest.

By considering the general case of n attributes and using an algebraic approach, we derive the precise sufficiency conditions under which two quality scales will be sensitive to the weights chosen. Note, however, that our conditions apply only to the case of two quality scales.

Beckwith and Lehmann's (1973) study of the importance of differential weights in a multiattribute model of consumer attitudes showed that such weights did no better than equal weights in predicting consumers' preference for various television shows. As the interattribute correlations increased, the differential weighting scheme decreased in importance. We will show, inter alia, why in a positively correlated environment such insensitivity to differential weights exists. Our theoretical results and empirical analysis show that quality scales *are* affected by the choice of weights, depending on the interattribute correlation environments. However, this sensitivity is not as arbitrary as suggested by Hjorth-Andersen (1984) or as direct as suggested by Curry and Faulds (1986).

In the remainder of this article, we present our theory and results. We then demonstrate our key results with two examples. The first example uses data obtained from *Test,* and the second example comes from *Places Rated Almanac*. We conclude with a discussion of the implications of our results for the construction of quality scales.

THEORY AND RESULTS

Let the quality for a series of K brands be judged on the basis of a set of N previously determined attributes. Then, a_{ik} is the score for the kth brand on the ith attribute, $i = 1, \ldots, N$, and $k = 1, \ldots, K$. We collect these attribute scores in the $N \times K$ matrix, $A = \{a_{ik}\}$. A quality scale across the set of brands is constructed by applying weights to the attribute scores. Let

$$\mathbf{w}_1' = (w_{11}, w_{12}, \ldots, w_{1N}), \tag{1a}$$

and

$$\mathbf{w}_2' = (w_{21}, w_{22}, \ldots, w_{2N}), \tag{1b}$$

be two weighting vectors with nonnegative elements. Then, overall quality scales $\mathbf{q}_1$ and $\mathbf{q}_2$ are constructed simply as the weighted linear composites (Curry and Faulds 1986; Curry and Menasco 1979):

$$\mathbf{q}_1' = \mathbf{w}_1'A, \tag{2a}$$

and

$$\mathbf{q}_2' = \mathbf{w}_2'A. \tag{2b}$$

The vectors $\mathbf{q}_1$ and $\mathbf{q}_2$ define the quality scales and contain the quality scores for each brand. The product-moment correlation between the two quality scales is

$$\mathrm{COR}(\mathbf{q}_1, \mathbf{q}_2) = \mathbf{w}_1'R\mathbf{w}_2/[(\mathbf{w}_1'R\mathbf{w}_1)^{1/2}(\mathbf{w}_2'R\mathbf{w}_2)^{1/2}], \tag{3}$$

where $R = \{r_{ij}\}$ is the $N \times N$ attribute correlation matrix that contains the interattribute correlations (Curry and Faulds 1986; Curry and Menasco 1979).

A correlation matrix must be either positive semidefinite or positive definite (Graybill 1976). Assume the attribute correlation matrix is positive definite. Then, the denominator of Equation 3 is always positive, and, hence, the sign of the correlation between the two scales depends only on the numerator of Equation 3. A little algebra (available from the authors) allows us to reexpress the numerator of Equation 3 as

$$\mathbf{w}_1'R\mathbf{w}_2 = \mathbf{w}_1'\mathbf{w}_2 + \sum_{i<j}\sum r_{ij}(w_{1i}w_{2j} + w_{1j}w_{2i}); \tag{4}$$

$i, j = 1, \ldots, N$.

Now, the following are always true: (1) $-1 \leq r_{ij} \leq 1$, and (2) for positive, standardized weights, $\mathbf{w}_1'\mathbf{w}_2$, the inner product of the two weighting vectors and the bracketed expression in the second term of Equation 4 are always positive. Equation 4 makes clear that the interaction between the interattribute correlations and the weighting vectors determines the sign of the correlation between the two quality scales. The bounded nature of the interattribute correlations allows us to determine specific sensitivity conditions.

Our approach is to construct a ratio based on the weighting terms in Equation 4 and compare it with the interattribute correlations. Let

$$RS = [\mathbf{w}_1'\mathbf{w}_2/\sum_{i<j}\sum (w_{1i}w_{2j} + w_{1j}w_{2i})]. \tag{5}$$

We call the RS ratio the index of relative strength. The numerator, being the inner product of the two weighting vectors, represents the main effect of the weighting schemes (since the ith term in the first weighting vector is multiplied by the ith term of the second weighting vector). The denominator captures the interaction between the two weighting vectors. If the main effect is high relative to the interaction, RS will be high. When the interaction of the two weighting schemes is high relative to their main effect, RS will be low.

Results

Our sensitivity conditions apply when the attribute correlation matrix consists of (1) all positive elements, (2) all negative off-diagonal elements, and (3) both positive and negative elements. In the first two cases, we compare the index of relative strength to the interattribute correlations to predict the sign of the correlation between two quality scales. For example, suppose all interattribute correlations are negative; if the index of relative strength is sufficiently high, the quality scales will correlate positively. However, if RS is sufficiently small, the quality scales will correlate negatively. When the attribute correlation matrix has a mixed structure, two results involving a modified RS are developed. We now derive the exact sufficiency conditions.

Let $r_{\max}$ and $r_{\min}$, respectively, be the nonzero maximum and minimum values of the off-diagonal elements of the attribute correlation matrix. The key results are I, IIb, and III. Result IIa is obvious. Proofs appear in the Appendix.

Result I. The following is a sufficient condition for COR ($\mathbf{q}_1$, $\mathbf{q}_2$) to be *negative*:

$$\text{All} \quad r_{ij} \leq 0, \quad i \neq j \quad (\text{i.e., } r_{\max} < 0),$$

and

$$RS < -r_{\max}. \tag{6}$$

Result II. Either of the following conditions is sufficient for COR($\mathbf{q}_1$, $\mathbf{q}_2$) to be *positive*:

$$\text{All} \quad r_{ij} \geq 0. \tag{7a}$$

$$\text{All} \quad r_{ij} \leq 0, \quad i \neq j, \quad \text{and} \quad RS > -r_{\min}. \tag{7b}$$

Result IIIa. Let R have a mixed structure. Also, let $r^{+}_{\max}$ be the maximum correlation among all the *positive* off-diagonal interattribute correlations and $r^{-}_{\max}$ be the maximum correlation among all the *negative* off-diagonal interattribute correlations. The following is a sufficient condition for COR($\mathbf{q}_1$, $\mathbf{q}_2$) to be *negative*:

$$r^{-}_{\max} < -[\mathbf{w}'_1\mathbf{w}_2 + r^{+}_{\max} \sum\sum_{\forall q<r \text{ such that } r_{qr}>0} (w_{1q}w_{2r} + w_{1r}w_{2q})]/[\sum\sum_{\forall s<t \text{ such that } r_{st}<0} (w_{1s}w_{2t} + w_{1t}w_{2s})]. \tag{8}$$

The above equation is simply a modification of the index of relative strength. This result shows that, when the maximum negative correlation among the attributes is sufficiently low, the correlation between the two quality scales will be negative. Such a result thus indicates sensitivity of the quality scale to the weights chosen.

Result IIIb. Let R have a mixed structure. Also, let $r^{+}_{\min}$ be the minimum correlation among all the *positive* off-diagonal interattribute correlations and $r^{-}_{\min}$ be the minimum correlation among all the *negative* off-diagonal interattribute correlations. The following is a sufficient condition for COR($\mathbf{q}_1$, $\mathbf{q}_2$) to be *positive*:

$$r^{+}_{\min} > -[\mathbf{w}'_1\mathbf{w}_2 + r^{-}_{\min} \times \sum\sum_{\forall s<t \text{ such that } r_{st}<0} (w_{1s}w_{2t} + w_{1t}w_{2s})]/[\sum\sum_{\forall q<r \text{ such that } r_{qr}>0} (w_{1q}w_{2r} + w_{1r}w_{2q})]. \tag{9}$$

Equation 9 shows that, when the minimum positive correlation among the attributes is sufficiently high, then the correlation between the two quality scales will be positive. Under such a condition, the quality scale will be insensitive to the choice of weights.

ILLUSTRATION

We first present an example using data obtained from *Test*. The data are from a 1983 study of 14 brands of stereo speakers. Each brand was evaluated on the three attributes of tone, technical performance, and durability, with weights of 0.55, 0.35, and 0.1, respectively. The attribute correlation matrix is as follows.

	Tone	*Technical performance*	*Durability*
Tone	1.00		
Technical performance	−.251	1.00	
Durability	−.017	−.172	1.00

Let $\mathbf{w}'_1$ = [0.55 0.35 0.1] be the original weighting scheme, and let $\mathbf{w}'_2$ = [0.1 0.35 0.55] be an alternative weighting scheme. Inspection of the attribute correlation matrix indicates $r_{\max} = -.017$ and $r_{\min} = -.251$; RS has a value of 0.303. Although all interattribute correlations are negative, RS is not less than $-r_{\max}$, so Result I does not apply. Since RS is greater than $-r_{\min}$ (.303 > .251), Result IIb is appropriate. According to this result, the correlation between the two quality scales should be positive. Substituting values into Equation 3 reveals that the correlation between the two scales, 0.402, is indeed positive.

This example satisfies the sensitivity conditions that Curry and Faulds (1986) generated: (1) all off-diagonal entries in the attribute correlation matrix are negative, and (2) weighting schemes are in the reverse rank order. Note, however, that the example fails to follow their result that the correlation between the two quality scales will be negative. Instead, these data satisfy our Result IIb and yield a positive correlation among the quality scales.

The second example is taken from *Places Rated Almanac* (Boyer and Savageau 1985), which, among other things, ranks 329 cities in the United States on a scale of overall quality. The index is computed with a unit weighting scheme based on the nine attributes of climate, housing costs, health care and environment, crime, transportation, education, the arts, recreation, and personal economic outlook. Becker et al. (1987) use these data to show that each of 134 cities can be placed in the first position simply by using different weighting vectors.

Let us now consider this data set in the context of our sensitivity conditions. Using the raw data from the 1985 edition of *Places Rated Almanac*, we calculated the interattribute correlation matrix among the nine quality of life attributes (see Table 1). The correlation

TABLE 1

CORRELATIONS AMONG ATTRIBUTES FROM CITY RANKINGS

	Climate	Housing	Health	Crime	Transport	Education	Arts	Recreation	Economic
Climate	1.00000								
Housing	−.24084	1.00000							
Health	.19603	−.40755	1.00000						
Crime	−.11475	.10190	−.15476	1.00000					
Transport	.07453	−.35490	.42262	−.25110	1.00000				
Education	.07942	−.19993	.44448	−.03921	.26585	1.00000			
Arts	.23004	−.48903	.71169	−.34142	.52829	.33502	1.00000		
Recreation	.12433	−.45306	.27133	−.28097	.37006	.09937	.47069	1.00000	
Economic	−.09263	−.32735	.08210	−.27708	.06748	.16131	.15527	.18617	1.00000

matrix is mixed in nature; 42 percent of the off-diagonal elements are negative. These data provide an excellent illustration of the sensitivity of an overall-quality scale to the choice of attribute weights. Using the weighting vector $\mathbf{w}_1' = [0\ 0.79\ 0\ 0\ 0\ 0\ 0\ 0\ 0.21]$, Becker et al. (1987) show that Sherman-Denison, Texas, can be ranked number one out of the 329 cities. Now, using the weighting vector $\mathbf{w}_2 = [0.9\ 0\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0]$, note how the rank of Sherman-Denison moves down to position 214!

Because the interattribute correlation matrix is mixed, Result III applies. From Table 1, we note that $r^-_{\max} = -0.03921$ and $r^+_{\max} = 0.71169$. Substituting the two weighting vectors above and the relevant correlations from Table 1 into the right-hand side of Result IIIa, we obtain a value of −0.0153. The left-hand side of Result IIIa is $r^-_{\max}$, which is equal to −0.03921. Making the required comparison, we note that this condition is satisfied, that is, $r^-_{\max}$ is less than the right-hand side of Result IIIa. From Equation 3, the correlation between the two quality scales, −0.22, is negative. Thus, as specified according to Result IIIa, the overall-quality scale is sensitive to the weights chosen.

DISCUSSION

The practical implication of our results is that, given an attribute correlation matrix, we can determine whether a particular choice of weights implies sensitivity of the quality scale. A reasonable extension of this study would be to derive the sensitivity conditions for m quality scales and n attributes. At present, however, we note that overall-quality scales that are derived from product categories whose interattribute correlations are all negative should be treated with caution. As Result I shows, different weighting schemes can cause such scales to correlate negatively. Calculating the index of relative strength and comparing it to $r_{\max}$ will indicate whether sensitivity is a concern.

Result II defines those situations in which a quality scale is not sensitive to the choice of weights. Result IIa specifies that two quality scales will always correlate positively when all the attributes are correlated positively (or zero). The obvious implication is that quality scales in *Consumer Reports*, with all attribute correlations nonnegative, are not sensitive to the choice of weights. However, as the weighting vectors move away from one another (i.e., as the inner product between them increases), the quality scales will tend to correlate less.

Results IIb and III generalize Curry and Fauld's (1986) conditions. Result IIb indicates that, even when all off-diagonal entries in the attribute correlation matrix are negative, two quality scales will still correlate positively as long as the index of relative strength is greater than the absolute value of the minimum off-diagonal interattribute correlation. In other words, in a negatively correlated environment, the rank order of the elements of two weighting vectors may not necessarily create sensitivity. Rather, it is the relative strength of their main effect versus their interaction, in conjunction with the minimum interattribute correlation, that leads to sensitivity.

Results IIIa and IIIb deal with mixed structures. These results indicate that the sensitivity of a quality scale depends on the number and the magnitude of positive and negative elements in the correlation structure derived from the two weighting schemes. This is an interesting and important result since it places no restriction on the exact nature of the mixture. As Curry and Faulds (1986) report, 20 percent of the consumer products in *Test* studied over a six-year period had all positive interattribute correlations, 2 percent were all negative, and 2 percent had zero interattribute correlations (i.e., $R = I$). The remaining 76 percent had mixed structures.

APPENDIX

Proof of Result I

Sign $\mathrm{COR}(\mathbf{q}_1, \mathbf{q}_2)$ = Sign $[\mathbf{w}_1'\mathbf{w}_2 + \Sigma\Sigma r_{ij}\ (w_{1i}w_{2j} + w_{1j}w_{2i})]$. Since $r_{\max} < 0$, this implies all off-diagonal elements of $R < 0$. Let us prove the result for the worst case, that is, when all off-diagonal elements of the R matrix are equal to $r_{\max}$. It is trivial to show that the result holds even when some of the off-diagonal elements are less than $r_{\max}$.

The value $\mathrm{COR}(\mathbf{q}_1, \mathbf{q}_2) < 0$, so by substitution,

$$\mathbf{w}_1'\mathbf{w}_2 + r_{\max} \sum_{i<j}\sum [w_{1i}w_{2j} + w_{1j}w_{2i}] < 0; \quad i, j = 1, \ldots, N.$$

Rearranging terms gives

$$\mathbf{w}_1'\mathbf{w}_2 < -r_{\max} \sum_{i<j}\sum [w_{1i}w_{2j} + w_{1j}w_{2i}]; \quad i, j = 1, \ldots, N.$$

And finally,

$$\mathbf{w}_1'\mathbf{w}_2/[\sum_{i<j}\sum (w_{1i}w_{2j} + w_{1j}w_{2i})] < -r_{\max}; \quad i, j = 1, \ldots, N.$$

Proof of Result IIb

Let all off-diagonal elements of R equal $r_{\min}$. It is easy to see that, if the result holds for this extreme case, the same result is achieved even when some off-diagonal elements are greater than $r_{\min}$.

Since $\mathrm{COR}(\mathbf{q}_1, \mathbf{q}_2) > 0$, this implies

$$\mathbf{w}_1'\mathbf{w}_2 + r_{\min} \sum_{i<j}\sum [w_{1i}w_{2j} + w_{1j}w_{2i}] > 0; \quad i, j = 1, \ldots, N.$$

Rearranging terms, we get

$$\mathbf{w}_1'\mathbf{w}_2 > -r_{\min} \sum_{i<j}\sum [w_{1i}w_{2j} + w_{1j}w_{2i}]; \quad i, j = 1, \ldots, N,$$

so that

$$\mathbf{w}_1'\mathbf{w}_2/[\sum_{i<j}\sum (w_{1i}w_{2j} + w_{1j}w_{2i})] > -r_{\min}$$

$i, j = 1, \ldots, N.$

Proof of Result IIIa

Let all the off-diagonal positive elements of R be equal to $r^+_{\max}$ and all the off-diagonal negative elements be equal to $r^-_{\max}$. The result holds even when some of the positive off-diagonal elements are less than $r^+_{\max}$ or when some of the negative off-diagonal elements are less than $r^-_{\max}$. Equation 4 can be rewritten as follows:

$$\mathbf{w}_1'\mathbf{w}_2 + r^+_{\max}[\sum_{q<r\,:\,r_{qr}<0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})] + r^-_{\max}[\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})].$$

The value $\mathrm{COR}(\mathbf{q}_1, \mathbf{q}_2) < 0$, so by substitution,

$$\mathbf{w}_1'\mathbf{w}_2 + r^+_{\max}[\sum_{q<r\,:\,r_{qr}>0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})] + r^-_{\max}[\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})] < 0.$$

Rearranging terms, we get

$$r^-_{\max}[\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})] < -\mathbf{w}_1'\mathbf{w}_2 - r^+_{\max}[\sum_{q<r\,:\,r_{qr}>0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})],$$

so that

$$r^-_{\max} < -\{\mathbf{w}_1'\mathbf{w}_2 + r^+_{\max} \times [\sum_{q<r\,:\,r_{qr}>0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})]\}/[\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})].$$

Proof of Result IIIb

Let all the off-diagonal positive elements of R be equal to $r^+_{\min}$ and all the off-diagonal negative elements be equal to $r^-_{\min}$. The result holds even when some of the positive off-diagonal elements are greater than $r^+_{\min}$ or when some of the negative off-diagonal elements are greater than $r^-_{\min}$. Equation 4 can be rewritten as follows:

$$\mathbf{w}_1'\mathbf{w}_2 + r^+_{\min}[\sum_{q<r\,:\,r_{qr}>0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})] + r^-_{\min}[\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})].$$

The value $\mathrm{COR}(\mathbf{q}_1, \mathbf{q}_2) > 0$, so by substitution,

$$\mathbf{w}_1'\mathbf{w}_2 + r^+_{\min}[\sum_{q<r\,:\,r_{qr}>0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})] + r^-_{\min}[\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})] > 0.$$

Rearranging the terms, we get

$$r^+_{\min}[\sum_{q<r\,:\,r_{qr}>0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})] > -\mathbf{w}_1'\mathbf{w}_2 - r^-_{\min}[\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})]$$

so that

$$r^+_{\min} > -\{\mathbf{w}_1'\mathbf{w}_2 + r^-_{\min} \times [\sum_{s<t\,:\,r_{st}<0}\sum (w_{1s}w_{2t} + w_{1t}w_{2s})]\}/[\sum_{q<r\,:\,r_{qr}>0}\sum (w_{1q}w_{2r} + w_{1r}w_{2q})].$$

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